Nodal Crossing Changes for Sun Synchronous Satellites with Solar Electric Propulsion

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The uses of low Earth sun synchronous orbits are outlined. A technique for changing the nodal crossing time for solar powered, electrically propelled satellites is discussed in detail, and its advantage over impulsive maneuvers is described. A detailed description of the method of calculating the mission time, propellant, and velocity increment requirements, given a particular spacecraft mass and propulsion system, is given for any nodal crossing change. This method includes the effects of loss of thrust during the time when the solar arrays are in Earth shadow, taking into account the change in shadow times as a function of nodal crossing time. A computer program to automatically calculate these mission parameters is described, and computer results are compared to the results of an earlier semiautomatic method.

Nomenclature

A	$=(1+h/R)^2$
a	= radius of circular orbit
α	= angle on Earth's disk defining x_e and y_e
β	= angle in orbit plane between thrust vector and tangent to orbit
γ	= angle between equatorial plane and the plane of the ecliptic
Δi	= angle of inclination increment
Δt	= increment of time
Δv	= velocity increment
$\Delta\omega$	= increment of ω
e	= orbit eccentricity
η	= thrust effectiveness
h	= spacecraft altitude above earth
i	= angle of orbit plane inclination to equatorial plane
m	= total spacecraft mass
μ	= Earth's gravitational constant
v	= orbit angular coordinate, complement of θ
ω	= nodal crossing time in degrees of arc
ω_e	= nodal crossing time in degrees where eclipses start
R	= Earth's radius
$\Delta\omega/\Delta t$	= time rate of change of ω
T	= total spacecraft thrust
t	= time spent in sunlight
T_e	= effective thrust, function of v
$<T_e>$	= effective thrust averaged over a revolution
θ	= orbit angular coordinate; $\theta=0$ or π at nodal crossing
V	= spacecraft orbit velocity
x	= dummy variable of integration for θ
(x_0,x_0)	= spacecraft Cartesian coordinates
(x_e,y_e)	= Cartesian coordinates where eclipses start

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Introduction

A SUN synchronous orbit[1] is one in which the orbit nodes, or equatorial crossing points, rotate eastward in an Earth centered coordinate system at exactly the angular velocity of the mean sun. Thus, the angle between the orbit plane and the mean sun line is constant, which is to say the local time at the nodal crossing points is a constant. The geometry is shown in Fig. 1, where it is seen that the nodal crossing point can be expressed as a time in a 24 h day or equivalently as an angle ω. For the present purposes, $\omega=0$ deg at a nodal crossing time of 6 p.m.

Sun synchronous orbits have certain properties that make them attractive for a variety of low altitude satellite missions.[2] Because a sun synchronous satellite crosses the equatorial plane at the same local mean sun time on each revolution, it can be used to view events on or above the Earth's surface at a preferred time of day to make the observation time or shadow conditions invariant. Certain passive radiation sensors also perform well only within a range of specific sun angles, or their seasonal output may be desired at a given local time.

Many sun synchronous missions have been identified for Western Test Range launch.[3,4] Reference 4 discusses a hypothetical Ion Propulsion Module (IPM) capable of capturing all 79 of these missions where only a few could be accomplished by existing or hypothetical conventional thruster modules used in impulsive maneuvers. The thrust strategy for the IPM study was a variation of the impulsive transfer reported in Reference 2, in which nodal crossing time changes were to be accomplished by making altitude changes to elliptical orbits at heights that would cause a nodal crossing drift due to the oblateness of the Earth. Then, when the desired nodal crossing time was reached, a two burn circularizing and altitude adjustment maneuver that would leave the spacecraft at sun synchronism was to be used. The IPM strategy[5] was to ascend quickly to the final altitude (in the 700 to 900 km range) and then to continuously thrust, with electric propulsion causing an orbit plane inclination change that would cause the needed nodal crossing change. Although more costly than the impulsive maneuver in terms of Δv, this strategy avoided altitudes where excessive atmospheric drag (low altitudes) or solar array degradation from radiation belts (high altitudes) would be present, and had the distinct advantage of being able to capture all identified missions.[4]

Deployment times were comparable to the conventional thruster methods.

The nodal crossing change technique, using low, continuous thrust from electric propulsion, is described in the following sections. The detailed description is given from the point of view of the mission planner who must calculate the time required for the maneuver, the propellant required, and the number of on-off cycles required, should the solar powered satellite pass through the Earth's shadow. A method of calculating the required velocity increment is also given.

To develop a method for making these calculations, initially the thrusting technique is described, first in the absence of complicating eclipses, and then with eclipses. This involves considering the thrust effectiveness, since it will be seen that as the satellite makes a revolution, the thrust varies sinusoidally in its effectiveness in causing the needed orbit plane change. Next, the detailed procedure for calculating the mission parameters is described. Finally, a computer program that uses the procedure is described, and recent results from it are compared to previously published results.

Effective Thrust

Effective Thrust with No Eclipses

For a circular orbit of altitude h above a spherical Earth of radius R the orbital velocity is

$$V = \left[\frac{\mu}{h+R}\right]^{1/2} \tag{1}$$

where the gravitational constant μ is equal to $3.98601 \times 10^{14} m^3/s^2$. The orbit inclination will change by an angle Δi measured in radians if a velocity component from a thrust T of $\Delta v = T\Delta t/m$ is applied perpendicular to the orbit plane at the nodal crossing point. That is, from Fig. 2

$$\Delta i = \frac{\Delta v}{V} \tag{2}$$

An equal but opposite Δv at the other node yields another Δi in the same sense. If the velocity increment is applied continously throughout a half-revolution as with electric propulsion, only the velocity increment gained at the node is totally effective. At the other two orbit extremes, the poles, the velocity increment causes no inclination change.

At orbital positions not at either pole and not at equatorial plane crossings as shown in Fig. 3, the $V + \Delta v$ resultant is based at a coordinate $a \cos\theta$ along the desired rotation axis in the equatorial plane, $\theta = 0$. Since the orbit is centered at the center of mass of the Earth which is in the nodal crossing plane at 0, and since the desired plane rotation is around the line $\theta = 0$, the thrust $T = m\ \Delta v/\Delta t$ is effective in rotating the orbit as desired only by the factor $T\cos\theta$. The effective average thrust that contributes to inclination plane changes is then

$$T_e = \frac{T}{2(\pi/2)} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta \tag{3}$$

Over each half-orbit from pole to pole, θ varies from $\pm\pi/2$ to $\pm\pi/2$ radians and

$$T_e = \frac{2T}{\pi} \tag{4}$$

which is to say that for inclination changes the constant, no eclipse thrust has an effectiveness of $2/\pi$. For the other half of the revolution after an ideal (instantaneous) thrust direction reversal at the orbit pole the same relations hold, so the total average effective thrust is given by Eq. (4). This thrust reversal also cancels out to first order the other average orbit change which is a rotation of the orbit plane around the nodal crossing rotation axis. This cancellation occurs each half-revolution because the thrust from node to pole is directed opposite to the thrust from pole back to the opposite node. For eclipse conditions, the same cancellation holds if the thrust is symmetric with respect to the nodes.

Effective Thrust with Eclipses

The effect of spacecraft eclipses as the spacecraft passes through the Earth's shadow is shown in Fig. 4. The figure is drawn for equinox conditions with the sun's rays that define the boundaries of the shadow in the plane of the paper. If the Earth were spherical, the orbit angular coordinate υ, which is

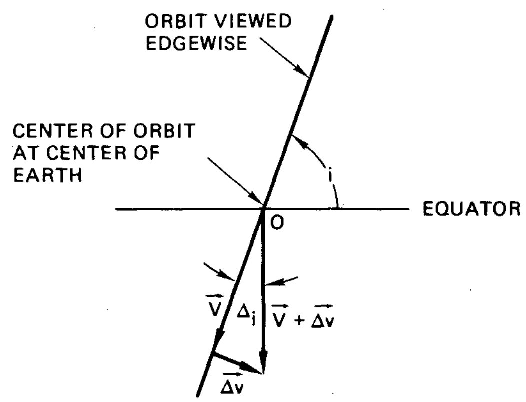

Fig. 2 Geometry of orbit inclination change. Axis of rotation is out of paper at 0.

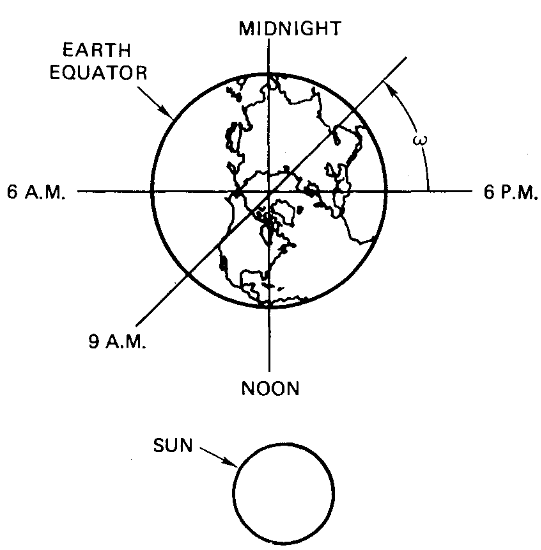

Fig. 1 Geometry of nodal crossing time.

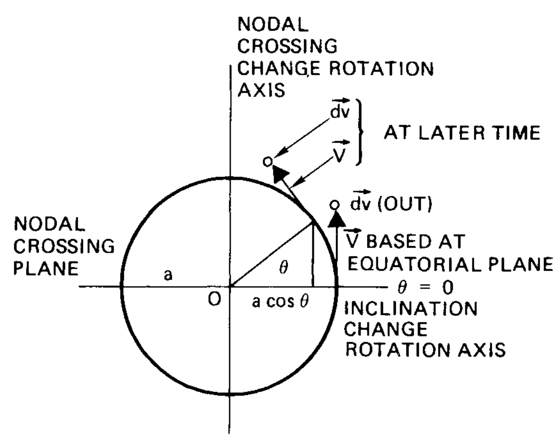

Fig. 3 Orbit viewed from the normal.

the complement of θ used in Eq. (3), is a function of the altitude and the nodal crossing time. For the orientation shown (noon nodal crossing time) υ is a minimum. The more general nonequinox case is covered later in this section.

If the nodal crossing time were slowly changed so that the orbit plane were to rotate about the polar axis, the triangle would leave the plane of the paper, and less and less of the orbit would be shadowed. The angle υ would increase from its minimum to its maximum, $\pi/2$, which occurs at some nodal crossing time which is to be found. Beyond that crossing time, υ is defined to be $\pi/2$, that is the orbit ceases to be eclipsed. It is necessary, therefore, to determine υ as a function of nodal, crossing time, altitude, and angle of inclination (ignoring the time of year for the moment). The derivation is made by using Figs. 5a-c. With the origin at the center of the Earth, and for an angle of inclination $i=90$ deg, the Cartesian coordinates at the equinoxes for the spacecraft are

$$x_0' = (R+h)\sin\upsilon \cos\omega$$

and

$$y_0' = (R+h)\cos\upsilon$$

where the sum of the Earth's radius R and the altitude h above the Earth's surface is the orbit radius. If the orbit plane is inclined to the equatorial plane at an angle i other than 90 deg, the geometry involves a rotation of the coordinate system through an angle of $\pm$ (90-i), so these equations become

$$x_0 = (R+h)\sin\upsilon \cos\omega \sin i \pm (R+h)\cos\upsilon \cos i$$

and

$$y_0 = \mp (R+h)\sin\upsilon \cos\omega \cos i + (R+h)\cos\upsilon \sin i$$

The coordinates on the Earth's surface where eclipses start are

$$x_e = R\sin\alpha$$

and

$$y_e = R\cos\alpha$$

Eclipses start when $x_0 = x_e$ and $y_0 = y_e$. Setting up these equalities, dividing through by R and squaring gives

$$\sin^2\alpha = \left[\frac{R+h}{R}\right]^2 (\sin^2\upsilon \cos^2\omega \sin^2 i + \cos^2\upsilon \cos^2 i \pm 2\sin\upsilon \cos\upsilon \cos\omega \sin i \cos i)$$

and

$$\cos^2\alpha = \left[\frac{R+h}{R}\right]^2 (\cos^2\upsilon \sin^2 i + \sin^2\upsilon \cos^2\omega \cos^2 i \mp 2\sin\upsilon \cos\upsilon \cos\omega \sin i \cos i)$$

Adding these equations eliminates α, and further factoring eliminates i to give

$$1 = \left[1 + \frac{h}{R}\right]^2 \left[\sin^2\upsilon(\cos^2\omega - 1) + 1\right]$$

Solving for υ gives

$$\upsilon = \sin^{-1}\left[\frac{1-(1+h/R)^2}{(\cos^2\omega - 1)(1+h/R)^2}\right]^{1/2} \tag{5}$$

So υ does not depend on the angle of orbit inclination i. But υ does depend additionally on the time of the year that the maneuver is performed, and that dependence is as follows.

The angle υ has been defined in Fig. 4 with respect to the equatorial plane which has been assumed to be the same as the plane of the ecliptic, that is, at the equinoxes. At other times the actual amount of useful thrusting time is less than that defined by υ if the thrust is to be kept symmetrical during the pole-to-pole eclipse half of each revolution. Such symmetry is necessary to avoid orbit perturbations, since the thrust effectiveness varies with the angular coordinate of the satellite in its orbit. Thus, in summer the thrusters should be turned off in sunlight before the start of each eclipse at an angle of $\upsilon+\gamma$ where γ is the angle between the equatorial plane and the plane of the ecliptic. They can then be restarted at the end of the eclipse to preserve symmetry. Similarly, in winter, the restart must be delayed. A maneuver not at equinox thus would have the effect of increasing the maneuver time, and the increase would be the greatest at the solstices where γ is about 23.5 deg.

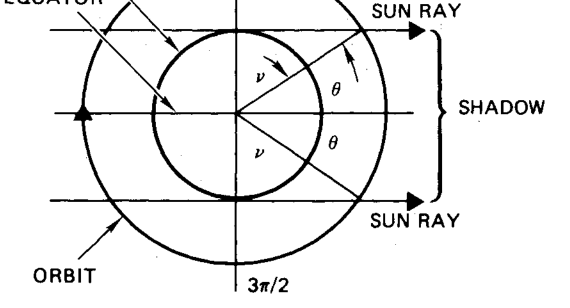

Fig. 4 Orbit geometry for noon nodal crossing time. When the spacecraft is at the triangle, the local time on Earth below it is noon.

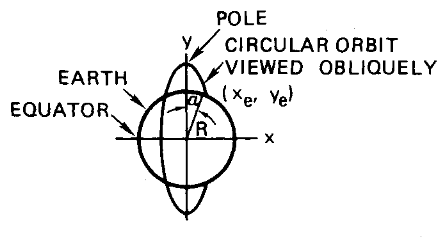

Fig. 5a View of Earth and circular orbit as seen from the sun.

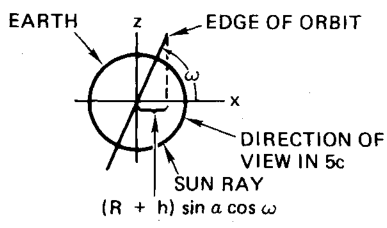

Fig. 5b View of Earth and circular orbit as seen from y axis.

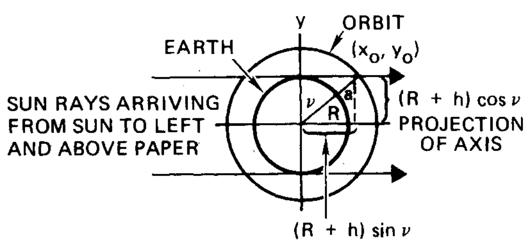

Fig. 5c View normal to orbit plane with $x\cos\omega$ as the projection of the x axis.

This effect of launch time has not been included in the calculation procedure or computer program presented here, but the necessary correction given the time of year of the maneuver is straightforward.

With Eq. (5) as is, that is, not including the time of year and using the constant

$$A = (1 + h/R)^2$$

and the definition of $v = \pi/2$ for all noneclipse ω's, the onset of eclipses occurs at

$$\sin v = 1 = \left[\frac{1-A}{A(\cos^2\omega_e - 1)}\right]^{1/2}$$

or at

$$\omega_e = \cos^{-1}\left(\frac{1}{A}\right)^{1/2}$$

or

$$\omega_e = \cos^{-1}\left[\frac{1}{1+h/R}\right] \tag{6}$$

For $\omega \geq \omega_e$, v is given by Eq. (5); for $\omega < \omega_e$, v is defined as $\pi/2$.

Now that the onset of eclipses is known by Eq. (6) and v is given by Eq. (5), the effective thrust can be calculated as was done for the no eclipse case that led to Eq. (4) from Eq. (3). The effective thrust over a complete revolution is Eq. (3) for the sunlit half plus a similar term for the eclipsed half. For the eclipsed half

$$T_e = \frac{T}{2v}\left[\int_{-\pi/2}^{3\pi/2+v} \cos x dx + \int_{\pi/2-v}^{\pi/2} \cos x dx\right] \tag{7}$$

where each term on the right is for one-quarter of the revolution that experiences Earth shadow. Integration yields the simple result

$$T_e = T\left(\frac{1-\cos v}{v}\right) \tag{8}$$

where again v is constrained to the maximum value of $\pi/2$.

For a typical altitude of 700 kM, the angle v and the quantity

$$\eta = \frac{T_e}{T} = \frac{1-\cos v}{v}$$

are plotted vs ω in Fig. 6. The ratio η may be thought of as a dimensionless quantity representing the thrust effectiveness for that altitude and nodal crossing time (or angle ω). Note that η combines the effect of thrust away from the optimum position of the orbit nodes as well as the effects of loss of thrust due to an eclipse.

Since electric propulsion implies very low thrust, only a tiny inclination change will result from one complete revolution of continuous thrust from pole to pole on the sun side of the Earth and shadow interrupted thrust during the other half revolution. Thus Eqs. (4) and (8) may be averaged over a revolution to give the total average effective thrust

$$\langle T_e \rangle = T/2\left(\frac{2}{\pi} + \frac{1-\cos v}{v}\right) \tag{9}$$

Calculation of Mission Parameters

For mission planning, the amount of time, propellant, and number of on-off cycles required to make a given nodal crossing change must be calculated. The calculation should cover the general case of a mission that has eclipses during some or all of the maneuver. The procedure is: first, the average nodal regression drift rate $\Delta\omega/\Delta t$ that the orbit would see at the midpoint of an incremental inclination change Δi is calculated. Second, Δt, the amount of time required to make that Δi change, is calculated using the average effective thrust from Eq. (9) with v chosen to represent a reasonable average value for the Δi increment. The average effective thrust is considered to act on the current average spacecraft mass for that increment. Third, $\Delta\omega$, the resultant change in nodal crossing angle is calculated as

$$\Delta\omega = (\Delta\omega/\Delta t)\Delta t$$

where $\Delta\omega/\Delta t$ depends on the current inclination angle. Fourth, a new spacecraft mass is calculated from the product of the propellant flowrate and Δt corrected for off time during eclipses. These steps are repeated until about one-half of the desired nodal crossing angle change has been accumulated by this drift technique. Then the process is continued with "reversed thrust" to change the inclination back toward the final desired inclination, and during this time, further nodal crossing angle changes are accumulated through drift. Hopefully, if the inclination change reversal point was correct, the final orbit of desired nodal crossing angle with proper inclination to give sun synchronism will have been obtained. If the nodal crossing angle change is too small (or large) when the final inclination is reached by calculation, the whole process must be repeated with a new inclination change reversal point beyond (or short of) the original one. Further iterations with increasingly adjusted inclination change reversal points are made until the desired sun synchronous orbit is obtained. The critical mission parameters of mission time and propellant consumption are then available as the sum of the Δt increments and the difference in beginning and ending spacecraft mass. The number of on-off cycles may be estimated by dividing the period for one revolution into the time spent in orbit while eclipses occurred. The period may be obtained using Eq. (1) and the fact that $2\pi(h+R)$ is equal to the orbit circumference.

Drift Rate for the Increment

The first step for each increment is to calculate an average drift rate or nodal crossing time change $\Delta\omega/\Delta t$. The required relation in degrees per second is[6]

$$\frac{\Delta\omega}{\Delta t} = -\frac{2.389744\times 10^9}{a^{7/2}(1-e^2)^2}\cos i \tag{10}$$

in which $a = R + h$ in km, the eccentricity e for a circular orbit is 0, and i is the angle of inclination. The drift rate is caused

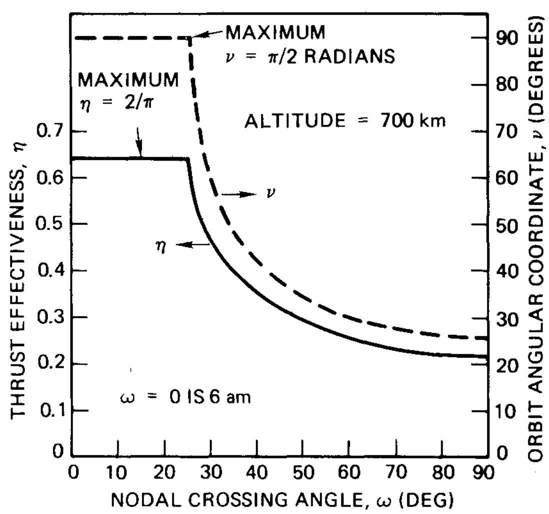

Fig. 6 Thrust effectiveness and orbit angular coordinate vs nodal crossing angle.

by the oblateness of the Earth, the so-called J_2 term in a detailed description of its gravity.[7]

Referring to Eq. (10), even for the largest nodal crossing change (3 h or 45 deg) that would probably be desired, the total required inclination change is only about 10 deg, so the Δi increments can conveniently be made very small, say 1 or 0.1 deg. It is, therefore, a very good approximation to use the angle of inclination at the start of the increment plus 1/2 Δi for i in Eq. (10) to calculate the average drift rate.

Time Required for the Increment

The second step of the calculation is to find the time required to complete the Δi change. From Newton's Second Law

$$\Delta t = m\,\Delta v / \langle T_e \rangle$$

and with Eqs. (1) and (2)

$$\Delta t = \frac{m\,\Delta i}{\langle T_e \rangle}\left[\frac{\mu}{h+R}\right]^{1/2} \tag{11}$$

in which $\langle T_e \rangle$ is given by Eq. (9). With a typical propellant mass flowrate of 10 to 20 mg/s the spacecraft mass m changes so little for a small Δi, which typically takes 10^6 s for a Δi of 1 deg, that it is a good approximation to use the spacecraft mass at the beginning of the increment. Inspection of Fig. 6 shows that both $\langle T_e \rangle = \eta T_e$, and v may change rapidly within a given increment. Therefore, care must be exercised for manual calculations. In the computer program described below, an iterative process to compare the averages of the actual initial and final η and v to the previously estimated average values first used was incorporated with a converging sequence to obtain and then use values good to an arbitrarily small amount.

Node Crossing Drift During the Increment

Once the above step is completed the resultant Δt is used with Eq. (10) to find the amount of nodal crossing change $\Delta\omega$ that has been gained during the Δi change.

New Spacecraft Mass

The spacecraft mass is then incrementally reduced by the product of the time spent thrusting and the propulsion system mass flowrate. The time spent thrusting is the time spent in sunlight and differs from Δt if eclipses are present. A good approximation for the sum of the thrusting periods is

$$t = \frac{2v + 180}{360}\,\Delta t \tag{12}$$

The reduced spacecraft mass is then used as the new mass for the next increment.

Incremental Calculations Then Iterate If Necessary

The above steps are repeated as outlined at the beginning of this Section to accomplish the total estimated Δi change up to the "thrust reversal" to make Δi incremental changes back to the final inclination angle. The sums of the Δt's and the $\Delta\omega$'s are the mission time and net nodal crossing change, respectively.

If a calculation is done for a mission that has no eclipses, then the thrust reversal should be at the point when one-half of the desired nodal crossing change occurs. With eclipses, however, the maneuver midpoint will not be the correct time to reverse the Δi changes. If a mission requires the spacecraft to go from full sunlight into eclipses or even from short eclipses into longer ones, the reversal of the Δi changes must be made before one half of the desired nodal crossing change is obtained, because the loss of thrust causes the time required to achieve a given Δi to increase which increases the amount of drift $\Delta\omega$ per increment. The converse of going from longer to shorter eclipses and/or into full sunlight similarly requires the proper reversal point to come after one half the nodal crossing change is reached since more available thrust lowers the Δt for a given Δi and thus lowers the amount of drift in $\Delta\omega$. Thus, with eclipses the reversal point will not be known initially and must be found by iterations of the whole procedure.

Spacecraft Starting Mass and Propellant Reserve

The above calculation procedure including iterations to find the proper reversal point necessarily requires that the dry spacecraft mass and propellant mass be known at the start of the maneuver. The former is well known, but the starting propellant mass including any desired reserve must be estimated prior to starting the calculation. After determining a set of iterated initial calculations based on this estimate, a second set of iterations will generally be required to adjust the starting propellant mass so as to accomplish the mission with the desired propellant reserve.

Combined Maneuver

Although the emphasis of this paper is the use of an electric propulsion system to make nodal crossing changes at a constant altitude, the thrusters might be used to make a small altitude change simultaneous with making part of the inclination change. Such a combined maneuver would be extremely effective.

To make such a combined maneuver, the thrust vector would be changed uniformly from a direction tangent to the orbit at the orbit polar positions to a direction perpendicular to the orbit plane midway between the poles. Thus, at any position of the orbit the component of the thrust T contributing to inclination change would be $T_{\Delta i} = T\sin\beta\,\cos\theta$ where the $\cos\theta$ factor is due as before to the loss of thrust effectiveness because of thrusting between the nodes, and β is the angle between the thrust vector and the tangent. The orbit position angle $\theta = \pi/2$ when $\beta = 0$ at one orbit pole, that is, $\theta = \beta + \pi/2$, so

$$T_{\Delta i} = T\sin^2\beta$$

Then

$$\frac{T}{x}\int_0^x \sin^2\beta\,\mathrm{d}\beta$$

is the average effective thrust as the position changes from 0 to x. Integrating over one orbit, $\beta = 2\pi$, gives the average effective thrust per orbit of $T/2$ which is lower than the value of $2T/\pi$ without the thrust vector change. But at the same time the $T\cos\beta$ component of T contributes to the altitude raising throughout the orbit with an average effective thrust

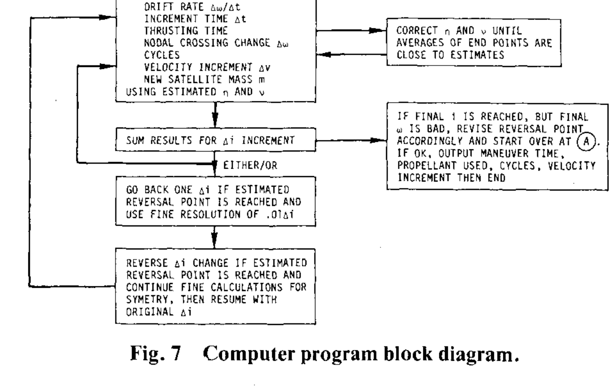

Fig. 7 Computer program block diagram.

over one orbit of

$$T_{\Delta h} = \frac{T}{\pi/2}\int_0^{\pi/2} \cos\beta \, d\beta = 2T/\pi$$

So the total effective thrust is $(0.5 + 2/\pi)T$ or about $1.14T$ during the part of the deployment that requires orbit raising.

Although very effective, the combined maneuver has limitations. It would require an additional degree of freedom to point the thrust vector, and probably only a small savings in mission time and propellant would be realized, since the combined maneuver portion of the mission generally would be only a small part of the total thrusting time.

Computer Program

A BASIC program was written for an Apple II computer to perform the above calculations automatically and more accurately. Figure 7 is a block diagram of the program. After the program inputs describing the spacecraft and the initial and final orbit characteristics are given, an initial estimate of the nodal crossing angle for the start of the reversal of Δi is made as one-half the desired nodal crossing change. Then, for the input Δi increment, the interim quantities, the drift rate, time to accomplish the Δi change, the resultant nodal crossing change, and the new spacecraft mass are calculated. At first, these interim calculations are made using the thrust effectiveness η and the eclipse angle v to be equal to the means of their new and original values. Then the calculations are repeated to get four new interim values and new mean η and v and so on, until the revised η and ν differ from their previous values by very little. The last values of Δt and $\Delta\omega$ are then summed, and the last mass is used for the next Δi increment.

If at the end of the current increment calculation the present estimate of the Δi change reversal point has been reached, the last summations of Δt and $\Delta\omega$ are deducted from their sums and the calculation process starts over from that point with a much smaller Δi increment, typically $0.01\Delta i$. This is done to gain very fine resolution of the reversal point and still retain the program speed of larger increments through most of the calculations.

The calculations then continue with the small increments until the reversal point is reached again. After reversal an equal number of small Δi increments are used to reach a larger even Δi increment, and then the program resumes with the large Δi's until the final inclination is reached.

Reversal Point Iterations

At this point if the final Δi calculation has been completed, and the final nodal crossing angle is within 0.001 deg of the desired angle, the results are output, and the program is completed. With eclipses this does not happen for some time. If the calculated change in ω differs from the desired ω by too much, the Δi change reversal point is revised accordingly, and a new iteration starts at Ⓐ in Fig. 7.

Propellant Reserve Iterations

After the iterated calculations to find the proper reversal point are completed, the entire procedure may have to be repeated with a revised starting propellant mass if the propellant reserve is not as desired. This revision is done manually with the present program.

On-Off Cycles

The program calculates the number of on-off cycles by storing the accumulated maneuver elapsed time spent under eclipsing conditions and dividing this by the revolution period. Because of the finer resolution of the program, the results are more precise than the former manual calculations.

Velocity Increment

The program also calculates the velocity increment, or Δv requirement, by summing over all the Δi increments the quotient of the amount of time spent thrusting during the increment divided by the current spacecraft mass for the increment, and then multiplying the quotient by the constant spacecraft thrust.

Computer Results

The computer program was applied to the four nodal crossing change maneuvers that were studied previously.[8] These four maneuvers are the deployment from, and the recovery to, the Shuttle of two separate spacecraft, designated A and B, which have requirements similar to TIROS and LANDSAT, respectively. In those hypothetical missions, both spacecraft were deployed from the same Shuttle and maneuvered to nodal crossing times three h earlier (Spacecraft A) and three h later (Spacecraft B). For retrieval, they were each maneuvered three h back together for rendezvous with another Shuttle.

Table 1 gives a comparison of the computer results to those of Ref. 4. To obtain a direct comparison the computer calculations started with the initial spacecraft plus propellant masses of Ref. 4. The former results were made by the first author using a hand calculator, and it is seen that those results were in surprisingly good agreement with the present, much more sophisticated computer results. The hand calculator results were obtained with rather large Δi increments of 1 deg corresponding to as many as 250 revolutions, and a very crude interpolation process was used to set the thrust reversal point somewhere within the mid-Δi increment. The computer results are superior, because they are obtained using Δi increments of 0.1 deg (~20 revolutions) except at around the reversal point, where they were 0.001 deg. This much higher resolution is especially important because the drift rate $\Delta\omega/\Delta t$ is at a maximum near the reversal point.

The program was also used to calculate the deployment and retrieval parameters of spacecraft A for a Δi of 0.01 deg and 0.001 deg near the reversal point. The results were within 0.25% of the computer results in Table 1, undoubtedly beyond the resolution of the process as a whole (see Conclusions). These calculations each took about 5-½ h on the Apple computer.

The quantities η and v for the effective thrust and start of eclipses were also determined much more accurately and precisely with the computer as described above. The hand calculator method was to estimate η and v with little or no iterative correction from a graph such as Fig. 6. Because of the shapes of the curves, this was a potential source of appreciable error.

Summary

The computer program has several large advantages over manual calculations, including the use of smaller Δi increments, better resolution of the mid-maneuver thrust reversal point, better determinations of the thrust effective-

Table 1 Results, Reference 4 compared with computer program

	Spacecraft A		Spacecraft B	
	Ref. 4	Computer	Ref. 4	Computer
Deployment time, days	140	137	141	143
Deployment propellant, kg	191	185	189	191
On-off cycles	946	939	990	1030
Velocity increment, m/s	–	2115	–	2163
Retrieval time, days	132	132	137	136
Retrieval propellant, kg	178	179	183	181
On-off cycles	924	905	980	989
Velocity increment, m/s	–	2212	–	2233

ness η and the start of eclipses at v, a more accurate estimate of the number of on-off cycles, and an accurate estimate of the velocity increment requirements. In addition, the computation time, not including propellant reserve iterations for each of the four maneuvers discussed, was reduced to about 35 min from about 12 to 16 h.

Conclusions

The accuracy and resolution of this calculation procedure, especially using the computer program, are adequate for a good first-order estimate of maneuver time, propellant usage, number of on-off cycles, and velocity increment required to make a nodal crossing time change using very low-thrust, solar powered electric propulsion. The resolution may exceed the magnitudes of several higher order forces such as atmospheric drag on the very large arrays, solar lunar gravitation, and the higher order forces due to the higher harmonics of the Earth's gravity potential.[10] As noted before, the procedure considers only the effects of the second harmonic, the so called J_2 term. Therefore, more resolution would be unwarranted without a consideration of other forces.

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References

[1] Wolverton, R. W., ed., *Flight Performance Handbook for Orbital Operations,* John Wiley and Sons, Inc., New York, 1963, pp. 2-222.

[2] Pritchard, E. I., "Standard Mission Destination for Sun-Synchronous Satellites Study," Aerospace Corporation Report ATR-79(7678-01) – 1, Contract NAS5-23572, Nov. 1979, pp. 1-7 and 2-4.

[3] Pritchard, E. I., "Standard Mission Destination for Sun-Synchronous Satellites Study," Aerospace Corporation Report ATR-79 (7678-01) – 1, Contract NAS5-23572, Nov. 1979, p. 2-4.

[4] Zafran, S., Gran, M. H., Fosnight, V. V., and Callens, R. A., "Ion Propulsion for Shared Shuttle, Sun-Synchronous Missions," AIAA Paper 81-0699, Section I, Las Vegas, Nev., April 1981.

[5] Zafran, S., Gran, M. H., Fosnight, V. V., and Callens, R. A., "Ion Propulsion for Shared Shuttle, Sun-Synchronous Missions," AIAA Paper 81-0699, Section III, Las Vegas, Nev., April 1981.

[6] Wolverton, R. W., ed., *Flight Performance Handbook for Orbital Operations,* John Wiley and Sons, Inc., New York, 1963, p. 2-235. The factor 1.407645×10^{16} in the last equation of Example 8 should be $(1.407645 \times 10^{16})^{1/2}$.

[7] Wolverton, R. W., ed., *Flight Performance Handbook for Orbital Operations,* John Wiley and Sons, Inc., New York, 1963, pp. 2-222.

[8] Zafran, S., Gran, M. H., Fosnight, V. V., and Callens, R. A., "Ion Propulsion for Shared Shuttle, Sun-Synchronous Mission," AIAA Paper 81-0699, Tables 5 and 6, Las Vegas, Nev., April 1981.

[9] Zafran, S., Gran, M. H., Fosnight, V. V., and Callens, R. A., "Ion Propulsion for Shared Shuttle, Sun-Synchronous Missions," AIAA Paper 81-0699, Section II, Las Vegas, Nev., April 1981.

[10] Wolverton, R. W., ed., *Flight Performance Handbook for Orbital Operations,* John Wiley and Sons, Inc., New York, 1963, pp. 2-240.